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Heuristics as beliefs and as behaviors: The adaptiveness of the "Hot Hand"

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Abstract

Gigerenzer (2000) and Anderson (1990) analyzed reasoning by asking: what are the reasoner's goals? This emphasizes the adaptiveness of behavior rather than whether a belief is normative. Belief in the "hot hand" in basketball suggests that players experiencing streaks should be given more shots, but this has been seen as a fallacy due to Gilovich, Vallone & Tversky's (1985) failure to find dependencies between players' shots. Based on their findings, I demonstrate by Markov modeling and simulation that streaks are valid allocation cues for deciding who to give shots to, because this behavior achieves the team goal of scoring more. Empirically I show that this adaptive heuristic is supported by the fallacious belief in dependency, more so as skill level increases. I extend the theoretical analysis to identify general conditions under which following streaks should be beneficial. Overall, this approach illustrates the advantages of analyzing reasoning in terms of adaptiveness.

Key Words: decision making, streaks, sequential information, hot hand, adaptive thinking

Heuristics as beliefs and as behaviors: The adaptiveness of the "Hot Hand"

Can the adaptiveness of a behavior be inferred from the validity of the belief that might generate it? The heuristic and biases approach to decision making embodied in the pioneering work of Tversky and Kahneman (e.g., Kahneman, Slovic, & Tversky, 1982; Gilovich, Griffin, & Kahneman, 2002) has focused on the validity of beliefs. Indeed, Gilovich (2002) in responding to an earlier version of the argument presented by this paper commented that "I don't see what is adaptive about the use of an outright fallacy." Yet one of the fundamental challenges that the adaptive thinking approach to decision making (e.g., Gigerenzer, 2000; Gigerenzer & Todd, 1999) appears to make is to focus on behavior and whether it attains goals, rather than the rationality of a belief. Often the link between the rationality of a belief and the adaptiveness of a behavior should be close, so the distinction has often been ignored. However to assume such a link is unjustified. The aim of this paper is to make this point by presenting an analysis of a behavior that can be shown mathematically to improve the outcome for a decision maker, and thus is adaptive. The fact that the behavior may be supported by a false belief is irrelevant to whether the behavior is adaptive or not, though the false belief may actually be beneficial to the extent to which it helps to maintain the adaptive behavior. Anderson (1990, p.33) made this point when he suggested that "there is no reason why normatively irrational heuristics cannot be adaptive" but it appears that concrete demonstrations of this point are required. The phenomenon I analyzed is the continuation of streaks of independent events, and a well-known example of this is the hot hand in basketball.

The Hot Hand Belief as Fallacy

My analysis of the hot hand behavior as adaptive is based on the evidence that the hot hand belief is false. So first it is necessary to explain the distinction between the behavior and the belief, and why the belief is regarded as a fallacy. Belief in the hot hand in basketball suggests that a player is more likely to hit a shot following a streak of successful shots, which implies that the probability of a player hitting a shot after two or three hits should be greater than after two or three misses. Gilovich, Vallone, and Tversky (1985) analyzed the shooting of a professional National Basketball Association (NBA) team over a season, and found that that the probability of success on any given shot was essentially independent of success on previous shots. From this empirical finding the conclusion has been drawn that the hot hand is a fallacy, and furthermore that it is potentially dangerous as it may lead to giving the ball to a player with a low probability of success.

Gilovich, et al. (1985) also presented evidence that basketball fans believe in the hot hand phenomenon. Despite the term "hot hand" often being heard on basketball telecasts, what exactly defines the phenomenon is unclear. So in order to determine if people with basketball experience believe in this phenomenon, Gilovich, et al. presented 100 basketball fans with a set of statements to endorse or deny. Two of the statements they gave were as follows, with percentages agreeing or disagreeing in parentheses:

(Statement 1) "Does a player have a better chance of making a shot after having just made his last two or three shots than he does after having missed his last two or three shots?" (Yes = 91%, No = 9%)

(Statement 2) "Is it important to pass the ball to someone who has just made several (two, three, or four) shots in a row?" (Yes = 84%, No = 16%)

(Exact wording from Tversky & Gilovich, 1989a, p. 20.) Gilovich et al. appeared to see these two statements, and others, as equivalent as they presented them as a list and then concluded that because there were high levels of endorsement for all questions, "basketball fans believe in streak shooting" (p. 298), which they consider equivalent to the hot hand. However, Statement 1 is about a *belief* that in itself has no behavioral consequences (although behaviors could be based on it), whereas Statement 2 is about a *behavior* (or a belief about a behavior) and thus has behavioral consequences in terms of who is likely to be given the next shot. Therefore the hot hand phenomenon can be seen as having two potentially separable components: there is a *hot hand belief* regarding dependence (as embodied by Statement 1) and a *hot hand behavior* of following streaks (as embodied by Statement 2). In the rest of the paper I will refer to either the *hot hand belief* or the *hot hand behavior* (or equivalently, *following streaks*).

Evidence that the hot hand belief is false

Gilovich et al.'s (1985) data addressed the validity of the hot hand belief, but not whether the hot hand behavior was beneficial. They analyzed the sequences of hits and misses by members of a NBA team (the Philadelphia 76ers). Their analysis encompassed data from an entire season (1980-1981) of home games (48 games, a total of 3800 shots across 9 players). In designing their analysis they did not deny that players experience streaks of hits and misses, or that some players hit a higher percentage of their shots than others do, instead they asked whether the observed streaks occur more than expected if each player's shooting success was a Bernoulli process with p (probability of success) being the player's shooting percentage over the season. For a Bernoulli process the probability of any sequence of events is determined only by p. Such a process is said to be *stationary* if p is constant over trials, which implies that events are independent. Belief by basketball players and fans in the hot-hand appears to suggest that basketball shots are not independent events, but instead the probability of hitting a shot varies with a player's recent shooting success. Therefore to test if the hot-hand was a fallacy, Gilovich, et al. tested the *nonstationarity hypothesis*, that a player's probability of hitting a shot varies over time and thus p is not a constant.

To provide multiple tests of the nonstationarity hypothesis Gilovich et al. (1985) analyzed the sequences of hits and misses for each player in their sample of shots. In none of their analyses could they reject the null hypothesis of stationarity. Furthermore they found no evidence of nonstationarity in free throw shooting, or for college players under controlled conditions. Given that the hot hand belief (as embodied in Statement 1) implies nonstationarity, they concluded that this belief was invalid. Although Gilovich et al. never tested the validity of the hot hand behavior (Statement 2), they imply that their analysis also established it as invalid when they suggest that the belief in dependence may be costly because it may lead players to pass to the wrong team-mate.

Later studies

Subsequent studies have largely supported Gilovich et al.'s (1985) failure to reject the nonstationarity hypothesis for basketball (Adams, 1992; Tversky & Gilovich, 1989a,b; Kass & Raftery, 1995), although there have been various attempts to try to refute their findings. Some of these have been based on idiosyncratic (Larkey, Smith, & Kadane, 1989) and refutable (see Tversky & Gilovich, 1989b) analysis, though perhaps more serious challenges have come from those who have questioned whether Gilovich et al's analysis had sufficient power to reject the null hypothesis of stationarity (Miyoshi, 2000; Sun & Tweeny, 2001). Others have found evidence of nonstationarity in other sports (Albert & Williamson, 2001; Gilden & Wilson, 1995; Klaasen & Magnus, 2001), however Tversky and Gilovich (1989a) emphasized that only conclusions about basketball shooting can be drawn from that data. The most impassioned challenge to Gilovich et al.'s result appears to come from professional basketball players and coaches who simple refuse to believe it (see Gilovich, 1991, p.17).

A further implication of Gilovich et al.'s (1985) finding is made clear by the analyses of Gilden and Gray Wilson (1995), Gilden and Wilson (1995) and Gilden (2001). Gilden and Gray Wilson made the point that evidence in favor of the nonstationarity hypothesis would not necessarily imply that successive events are dependent. Independent events can be generated by nonstationary Bernoulli processes if changes in *p* are independent of the outcome of events. Thus fluctuations in *p* arising from learning, oscillation in performance, or stochastic change would all produce evidence of streakiness by Gilovich et al's criterion, although with characteristic patterns. A concrete example of this is Clark's (2003) finding that the evidence he found for streaks amongst professional golfers for hitting at or below par was due to variations in course difficulty rather than the players themselves being streaky. Thus Gilovich et al.'s finding that the professional players' probabilities of hitting shots appears to be constant, does not just argue against the hot-hand as implying dependencies between shots, but it also implies that there is no fluctuation in player's probabilities of hitting shots over an entire season. In some ways, this implication of Gilovich et al's data is more surprising than the implication that shots are independent events, given that players face different defenses and match-ups from night to night, as well as possibly becoming injured or fatigued over a long season.

Gilovich et al. (1985) interviewed too few professional players to be confident that they hold the hot hand belief, rather than it just being a belief limited to basketball fans. Therefore Tversky and Gilovich (1989b) attempted to find evidence of the hot hand belief amongst professional players by examining whether a player was more likely to take the team's next shot after a hit than a miss. To do so, they analyzed the conditional probabilities of a player taking the team's next shot. Averaging over 18 NBA players, they found the conditional probability of a player taking the next shot after a hit was .25, but after a miss it was only .20. Note however that these data indicate that professional players have a bias to follow the hot hand *behavior*, but did not directly indicate that they hold the hot hand *belief*. Tversky and Gilovich's conclusion that their data were evidence that NBA players hold the hot hand belief relied on the assumption that there is no other way to explain this behavior.

Implications

Gilovich et al's (1985) results have stood up to many challenges in the years since their publication and they have been cited in over 100 journal articles, according to the ISI Social Sciences Citation Index database. Many of these citations are in the decision-making literature, but there are also many citations in sports science (e.g., Vergin, 2000) and economics (e.g., Pressman, 1998). Their influence has spread to the literature on law (Hanson & Kysar, 1999) and even religion (Chaves & Montgomery, 1997). One reason for the wide interest in Gilovich, et al.'s result appears to be the implications it seems to have for behavior when the empirical result regarding the hot hand belief is extrapolated to the hot hand behavior. As Gilovich, et al. pointed out, belief in the hot-hand may have consequences for how players conduct the game, and the data from Tversky and Gilovich (1989b) seem to show that this is true in that players are more likely to take the next shot after a hit than a miss. However basketball is not the only task in which people have been observed to act in ways suggesting that they give added weight to recent streaks of events. In the stock market *momentum traders* buy stocks that have recently increased in value (Johnson, 2002). In many sports there is a belief that a wining team has momentum and this leads gamblers to favor such teams (Camerer, 1989; Lee & Smith, 2002). Kanwisher (1989) pointed out that belief in streaks underlay the "domino theory" which formed the basis of American foreign policy for a number of years. In the economics and finance literature the basis of many of the references to the hot hand are its possible behavioral implications. For example, Camerer (1989) states that, "The important question for economics is whether mistaken beliefs like the hot hand fallacy make allocation of resources suboptimal." (p. 1257) Misallocation of resources is important to society; so the seemingly practical implication of the hot-hand fallacy have even made it into *Money* magazine, where Willis (1989) drew from Gilovich, et al.'s study a warning against using past performance to choose mutual funds (whether empirically there is a hot-hand for mutual fund managers has become a matter of debate, see Carhart, 1997; Elton, Gruber & Blake, 1996; Hendricks, Patel, & Zeckhauser, 1993; Metrick, 1999). However, the existence of streak, or momentum, effects in the stock market has become well established and is leading to vigorous debate over the reasons why following streaks can produce abnormal profits (e.g., Jegadeesh & Titman, 1993, 2001).

The importance of streaks to people has recently been reinforced by brain imagery work by Heuttel, Mack and McCarthy (2002) that demonstrated that different areas of the brain are more activated by streaks than by non-streaks. Not only do specific areas of the brain react to streaks, but the strength of the signal is related to the length of the streak. If the brain is wired to notice streaks, then it is unsurprising if it is also found that people utilize streaks in making choices. Furthermore it also implies that doing so is probably useful in some way.

It is not impossible that some future analysis of basketball shooting might convincingly reject the null hypothesis of stationarity and thus revise the assessment of the hot hand belief, but this paper is not only about what happens on the basketball court. Instead my analysis will focus on what behavioral implications flow from the empirical evidence that the hot hand belief is a fallacy. In doing so, I will demonstrate that invalidating a belief does not necessarily invalidate the behavior that might be based on that belief. Tversky and Kahneman (1974) convincingly demonstrated that people make reasoning errors, yet Gigerenzer (1996) in his argument with Kahneman and Tversky (1996) over the validity of the interpretations given to that data, stated "I believe this debate is fundamentally about what constitutes a good question and a satisfactory answer in psychological research on reasoning." (Gigerenzer, 1996, p.592) Whether the hot hand behavior is adaptive is a different question to that which has been asked before, and I will show that the answer to this question leads to a different and more intriguing conclusion regarding the hot hand phenomenon: Gilovich et al.'s empirical data imply that the hot hand behavior is adaptive.

Plan of the Paper

The aim of this paper is to demonstrate how the hot hand behavior is adaptive, to examine what general implications this analysis has for understanding people's reactions to streaks, and more broadly what implications this has for different approaches to decision making. Therefore the remainder of the paper is divided into five sections. The first section explains the adaptive approach to decision making and what it means to say that the hot hand behavior is adaptive. The second section supports the claim that the behavior is adaptive by showing this must be true if Gilovich et al's (1985) data are accurate. Most importantly, this is done by developing a Markov model of basketball shooting. In the third section this model is generalized to sequences of choices so as to determine the conditions under which following streaks should be adaptive. From these are generated empirical predictions regarding the conditions under which people will be likely to follow to streaks, and some empirical evidence regarding these is described. The implication of this analysis is that a belief in streaks may arise because following streaks is adaptive, so in the fourth section is reported a study of people's attitudes regarding the hot hand belief in basketball and the hot hand behavior. This study examined the connection between the belief and the behavior as a function of basketball experience. The final section discusses how treating the hot hand as a belief or a behavior highlights the importance of a critical difference between the different approach to decisions making: that what is normative is not necessarily what people should do. The heuristics and biases approach assumes that it is, but adaptive approaches do not. Taking an adaptive approach casts a very different light on streaks, one that opens up the study of many interesting phenomena.

The Hot Hand Behavior as Adaptation

Gigerenzer and Todd (1999) emphasized that humans and animals make decisions about their world within the bounds of limited time, knowledge, and computational power. So they propose that much of human reasoning uses what they term an *adaptive toolbox* containing fast and frugal heuristics that fit to these limitations. They suggest that people's thinking is adaptive to the environment they are faced with, therefore the main purpose of heuristics is to reach goals. Similarly, Anderson (1990) emphasized analyzing cognition in terms of behavior that achieves goals, an assumption fundamental to his ACT-R framework for cognition (Anderson & Lebiere, 1998). He proposes that "The cognitive system operates at all times to optimize the adaptation of the behavior of the organism." (Anderson, p.28) Thus it is irrelevant whether the beliefs that these heuristics are based on fit to the statistical norms that have been used to define rational thinking. Instead Gigerenzer's and Anderson's approaches set the criterion for the evaluation of heuristics as being whether they produce beneficial behavioral outcomes, where beneficial is defined in terms of the attainment of goals.

What is adaptive for the members of a basketball team? Assuming the goal is to win, then it is adaptive to try to maximize the number of points the team scores when the members of the team have possession of the ball. From this point of view, the most relevant question to ask regarding the hot-hand phenomenon is: does the behavior of having a bias to give shots to a player experiencing a streak help attain the team's goal of scoring as many points as possible?

Every time a basketball team has possession of the ball, the members of the team individually and collectively face the question of who should take the next shot. Unless a time-out is called, a shot in an NBA game must be taken within 24 seconds of a team receiving possession, but usually a player (or coach) must make a decision about who to allocate the next shot[1] to in a matter of a few seconds at most. For this allocation decision there are probably a number of allocation cues that are valid and may be used by players. One piece of information that every player on a NBA team may be aware of are his teammates' career or season shooting percentages (i.e., what percentage of their shots players have previously hit). There is little doubt that this is a valid allocation cue, but shot allocation is not a single-cue decision otherwise the same player would take every shot. There are undoubtedly allocation cues concerning the positioning of teammates and opponents, but Tversky and Gilovich's (1989b) finding that players are more likely to get the next shot after a hit suggests that recent success is also utilized as a cue. Given that allocation is a multi-cue decision process, are streaks not only a utilized cue but also a valid allocation cue that should be given weight in the decision making process? If it can be shown that giving positive weight to such a cue will increase scoring behavior then such a cue is valid.

Direct empirical data addressing the cue validity of streaks in basketball is unlikely to be forthcoming as it would be difficult to manipulate professional players' belief in the hot hand, and any correlation between belief in streaks and team scoring would be fraught with potential biases. (For example, belief in the hot-hand might be correlated with other beliefs that either negatively or positively affect scoring, in which case belief in the hot-hand may be correlated with scoring for irrelevant reasons.) Fortunately, the data of Gilovich et al. (1985) and Tversky and Gilovich (1989b) provide the necessary evidence to answer this new research question.

My basic argument is fairly simple to state. If one accepts Gilovich et al.’s (1985) data showing that basketball shots are independent events, then the better a player's underlying ability to hit shots is, the larger the number of runs of hits that player will have. Thus streaks are information. The straightforward mathematical intuition behind this can be seen in another context in Kareev (1995). Streaks and shooting percentages are separate cues, yet both will lead to more shots being allocated to players with higher *true likelihoods* of hitting a shot. A critical point here is that players' historical base-rates for hitting shots, as represented by their observed shooting percentages, are imperfect indicators of their true likelihoods of success at a given point in time, except under very specific circumstances. The relative usefulness of base-rates and streaks as indicators of true likelihoods will depend on the conditions under which a decision is made. I will later discuss the narrow conditions under which shooting percentages are the only cue necessary for determining a player's true likelihood of success. Such conditions are unlikely to be found on basketball courts, or in many other situations outside of the casino or the psychology laboratory. Under all other conditions, if successive events are independent then using streaks as part of a multi-cue decision process will increase success. Simply put, streaks convey information and such information can be utilized.

Allocation cues

To state that something is invalid is an incomplete statement, as it has to be invalid (or valid) with respect to something. With respect to behavior, Gilovich et al. (1985) showed that the hot hand belief was invalid for predicting *when* an individual will be most likely to hit a shot. In this sense it is an invalid *individual timing* cue. However the validity or invalidity of the hot hand behavior (i.e., give the shot to the player on a streak) seems best assessed with respect to whether streaks are a valid or invalid *allocation* cue for the behavior of *who* to give the next shot to.

The distinction between cues for individual behavior (i.e., when will a given player hit a shot?) and cues for allocation between options (i.e., who should take the next shot?) is critical to the argument presented in this paper. Without this distinction, Gilovich's (2002) argument would be correct in that I would be claiming that something is simultaneously valid and invalid. Yet there is no contradiction in arguing that something may be invalid as a cue to individual behavior, but a valid cue for allocation between individuals. For example, consider the classic research by Estes (1954) and others into probability matching. The research into probability matching used a task in which the participant had multiple options that had different underlying probabilities of success (e.g., a two-armed slot machine for which the payoffs could differ). On each independent trial, the participants had to choose which of the options to take, and then they could observe the outcome. It was found that people will learn to chose the option with the higher base-rate of success, but that they behaved nonoptimally in that they choose that better option with a probability equivalent to its success rate rather than on every trial. Note that base-rates of success are valid allocation cues in this situation, yet invalid individual cues for when an option will pay-off. If we tested Gilovich et al's (1985) Statement 1 on such data, we should find that events are independent (i.e., despite a higher base-rate for one option, it is not more likely to pay-off after a success than after any other result). Yet if we tested Gilovich et al.'s Statement 2 on this data (i.e., should you choose Option 1 more often if it has a higher base-rate?) we would find that base-rate is a valid allocation cue, and ignoring this cue would be labeled an error. Thus invalid individual cues may be valid allocation cues when assessed against the relevant criteria.

Assessing Streaks as Allocation Cues

I will address the question of whether streaks are valid allocation cues in three ways: 1) testing the implication that Gilovich et al's (1985) data should show that streaks are associated with higher shooting percentages; 2) by building a Markov model that includes a hot hand parameter; 3) by computer simulations showing that a team that follows the hot hand behavior will outscore one that does not (i.e., one that gives a hot hand parameter the value zero), everything else being equal. The claim I will make is not that the hot hand is the only cue – or even necessarily the best single cue – for who to allocate the ball to, but simply that a streak is a valid allocation cue that if given a positive weight will increase scoring over and above the contribution made by whatever other cues are being used. I will not present new data to support my analysis; instead I will use Gilovich et al.'s data to support my claims and to justify my assumptions. Thus this paper does not challenge Gilovich et al's data, but instead I will show that different conclusions flow from their data than those that have normally been assumed.

Streaks and Shooting Percentages

Gilovich et al. (1985, Table 1) presented the probabilities of players on the Philadelphia 76ers making a shot after runs of hits or misses of length 1, 2 and 3, as well as the frequencies of each run for players. My Table 1 was derived from these data by calculating for each player the proportions of his total number of shots (excluding a player's first shot in a game) which were parts of runs of hits or misses of length 1, 2, and 3.

If shots are independent events, then the higher a player's shooting percentage, the larger the number of runs of hits and the fewer number of runs of misses that player should experience. Table 1 presents the correlations between a player's shooting percentage and the proportion of his shots that were parts of runs of each length. Whereas the .99 and -.99 correlations for 1 hit and 1 miss respectively are trivial (they would have magnitude 1.0 if the first shot in a game was not ignored), the high correlations between 2 hit/miss and 3 hit/miss and streaks (all $p < .01$) are not trivial. These correlations should only exist if streaks are strongly associated with shooting percentage, and given that Gilovich et al.'s (1985) analysis indicates that there are no fluctuations in shooting percentage, these correlations suggest that streaks are valid predictors of underlying likelihood of success. Whereas these data empirically demonstrate that the information conveyed by streaks was predictive of shooting percentage (which are indicators of underlying ability), they do not tell us anything that could not be concluded from Gilovich et al.'s analysis. Given that Gilovich et al.'s data demonstrated that shots are independent events and that shooting percentages do not fluctuate, it must be true that streaks predict shooting percentage. If this analysis did not produce the above results then it would be equivalent to showing that Gilovich et al. were incorrect in claiming that there are no more streaks in the data than would be expected by chance. Thus further analysis is necessary to demonstrate the implications of their data.

A Markov model

To show that streaks are a valid allocation cue, I constructed a Markov model of the first two shots in a game. This model uses the assumptions that Gilovich et al's (1985) data showed applied to basketball, although my model applies to any situation which fits to the same assumptions: that a choice must be made between alternatives that have fixed probabilities, and that the success of an option is independent of its recent success. (I will later relax these assumptions when trying to generalize this model.)

The model has four parameters, all of which can take any value between 0.0 and 1.0, and it represents a Player X and a Player Y. Although this seems to assume only two players, there is no reason Player Y could not represent all the four players who are not Player X, or other groupings of players. A bias parameter b is the base-line probability of giving the next shot to Player X, whereas the same probability for Player Y is $1-b$. The b parameter represents any bias players may have to give the shot to a player (e.g., high shooting percentage, perceived ability, friendship, etc) that could be applied independently of any factors derived from recent streaks of success. The model has separate parameters for the shooting ability for the two players, x and y for Players X and Y respectively. The streak parameter h temporarily elevates the probability of a player being given the next shot after a hit, and thus represents the weight given to streaks as an allocation cue. Thus the probability of Player X being given the next shot after a hit is $b + h(1 - b)$. Obviously when h=0 there is no change in the probability of the player receiving the next shot, which represents the team having no belief in the hot hand. If h=1 then the probability that the player who just hit being given the next shot equals 1.0, so a player will always receive the next shot after a hit. The model does not incorporate a parameter for belief in a cold-hand because there is no empirical evidence that people hold or act on such a belief, thus we can reduce the number of parameters the model must have.

The full model is shown in Figure 1, and from this was calculated the expected number of hits from the first two shots by summing the expected number of shots of the sixteen possible states after two shots. There are 16 possible states because there are two branching points for each shot, who gets the shot and what was the result of the shot. The first two shots of a game are used because there has to be at least one shot before there can be a streak. If the hot hand can be shown to increase expected scoring over the first two shots then it must do so over any number of shots, if shots are independent.

Equation 1 is the expected number of hits after two shots, calculated by summing the expected outcomes of all 16 states in Figure 1 (See Appendix for the derivation):

(1) $$\text{E(hits after two shots)} = 2(b(x - y) + y) + h(b - b^2)(x - y)^2$$

The most significant implication of Equation 1 is that the $h(b - b^2)(x - y)^2$ component can *never* be negative given that all parameters have a range of 0 to 1, yet this is the only place h appears in the equation. Thus giving a positive value to the streak cue can never lower the expected number of hits. Instead, any positive value of h will raise the expected outcome except under two specific conditions: h will have no effect when $x = y$ (i.e., if there is no difference between players then it does not matter how shots are allocated), and when $b=1$ or $b=0$. Equation 1 will have its maximum value when $b=1$ if $x > y$, however, setting b to either 0 or 1 represents a pure strategy. These pure strategies are neither observed when people choose repeatedly between options with different probabilities (even when this is optimal), nor would they be desirable in NBA basketball. For any other given values of *b*, a positive value of *h* (i.e., $0 < h$ 1.0) will raise the expected outcome, so having some bias to use streaks for allocating shots must increase expected scoring.

However the fact that Equation 1 is at a maximum when $b=1$ (and $x > y$,) points to a simplifying assumption of the model, that probabilities are stable. This assumption held in Gilovich et al's (1985) data but is unlikely to always be valid. I will address this issue in the section of the paper on the general conditions under which streaks are a valid allocation cue. Overall, the model shows that for anything other than these exact values of *b*, a positive value for *h* (the streak cue) increases scoring. Thus when players are more likely to allocate a shot to a player who has just hit a shot, they are behaving adaptively, even though this bias appears to only be small, an increase in probability of getting the next shot from .20 after a miss to .25 after a hit according to Tversky and Gilovich (1989b).

The advantage to be gained from the small bias to follow streaks in NBA basketball should not be large. A rough calculation can be done if we assume that $h = .25$ (the proportional increase in the chance of getting a shot after a hit, based on Tversky & Gilovich, 1989b), and that *x* and *y* are the maximum and minimum shooting percentages in Gilovich, et al.'s (1985) sample of NBA players (.62 and .46, although there may be specific reasons why these particular players had these shooting percentages, my aim here is simply to utilize plausible indicators of shooting ability). The highest percentage of all shots taken by one player in their data was 23.3%, so as a two player bias that would mean $b = .54$. Using these parameter values, $h(b - b^2)(x - y)^2 = .0020$. The average number of shots per game taken by the Philadelphia 76ers in Gilovich, et al.'s data was 79.2, thus the increase in expected number of shots hit per game when there is belief in the hot hand would be .16. So maybe every seven or eight games, a team that believes in the hot hand might score an extra basket than one that did not. This is only a small advantage, but a large number of basketball games are decided by two points or fewer, so I suspect a team would take this advantage if it was offered to them. However, I will later discuss how the conditions that apply to NBA basketball might make it a situation in which the hot hand behavior would help less than it might in many other situations, including those in which NBA players first acquired their skill.

What defines a streak? Figure 1 presents a model in which the hot hand is defined as a single hit, but what generally defines a streak is not clear. Maybe it is three hits is a row, maybe at least four out of the last five hits, maybe it is a function of how many hits in a row a player has made. A feature of the model in Figure 1 is that whatever definition of a hot hand is used, then an advantage proportional to $h(b - b^2)(x - y)^2$ would be expected. As long as recent success temporarily raises the probability of giving shots to a player experiencing it, then how that run of success is defined does not matter. Shot 1 in Figure 1 could simply be the final event in a sequence that constitutes a streak and triggers a non-zero value for h. For a sequence to be called a "streak" only seems to imply more success than normal and that the final triggering event is a hit, which are conditions that would lead to a higher expected score when h is assigned a value greater than zero in Equation 1. Similarly, making h a function of the number of hits in a row would simply vary the magnitude of h, and thus not fundamentally change Equation 1.

Other cues. There is little doubt that there are cues not incorporated into Figure 1 that are used during basketball games and that influence who is allocated the next shot. For example, different players appear to have different shooting abilities depending on where they shoot from. One way of modeling this would be to conditionalize the b and h parameters on where a shot is to be taken from. So if a team needs a three-point shot then who is allocated the shot should be influenced by who has historically hit a large percentage of their three-point attempts, and who has hit their recent three-point attempts. Little or no weight may be given to the fact that a player hit their last shot under the basket. If such a model was implemented, then positive conditionalized values of *h* and *b* should still result in more hits. Interestingly, Gilovich, et al.'s (1985) data argues that individuals taking shots with different underlying probabilities of success must happen less often than one would expect, as the data suggests that observed shooting percentages are constant. Thus either these other allocation cues have little effect on shooting percentages, or players are very efficient at using those cues such that they do not give shots to players when they are under conditions detrimental to their underlying ability to hit shots.

Defensive pressure. In suggesting that belief in the hot-hand may be detrimental, Gilovich, et al. (1985) point out that the defense may exploit this belief by concentrating on the "hot" player. Game theory argues that this could be the weakness of any pure strategy, which is why many competitive games are at equilibrium with a mixed-strategy (see Rapoport, 1966). The Markov model in Figure 1 does not explicitly represent defensive pressure, so it is worth considering how defensive pressure could be modeled in a way that is consistent with the results of Gilovich et al.

One way to model defensive pressure would be to assume that the defense makes it harder for players who hit their last shot to hit their next shot. However, this would mean that a player has a temporarily lowered likelihood of success just after hitting a shot, which would imply that basketball shots are not independent events. We know from Tversky and Gilovich (1989b) that players are more likely to take the next shot after a hit than a miss, yet the data of Gilovich et al. (1985) showed that players' shots are independent events. Therefore to model defensive pressure as a temporary lowering of shooting success would violate the empirical data. (However, in order to generalize this model beyond basketball, I will later consider the implications of sequences in which successes are negatively correlated.)

A way to model defensive pressure without violating the constraints imposed by the existing data would be to assume that defensive pressure on the player who hit his last shot makes it harder for that player to be given the next shot. This could be modeled by creating a new parameter that represents a temporary downward adjustment to the probability of a player being allocated the next shot, in effect a decrease in the h parameter. However as long as h remains positive, then the offense should do better even if both the offense and defense act on an expectation that streaks should be followed. Negative values of h could be allowed, but Tversky and Gilovich’s (1989b) evidence that players are more likely to take the team's next shot after a hit than a miss, suggests that the net effect is not negative, at least for that sample. Therefore it appears that if one tried to add defensive pressure to the model while staying within the constraints imposed by the existing data, then the hot-hand behavior would still be expected to produce a better outcome for the offense.

A Computer Simulation

The implications of Equation 1 can be made more concrete with a computer simulation of the effect on team scoring of assigning a nonzero value to the streak parameter h when holding constant players' shooting abilities and the bias to give the shot to one of the players. It is difficult to visualize the effect of four independent parameters on an output measure, so two simplifying assumptions were made. First, only x, the shooting ability for Player X will be varied. The shooting ability for Player Y (y) will equal $1 - x$. Second, only two values of h will be used, $h=0$ (i.e., never follow streaks) and $h=1$ (always follow streaks). Equation 1 shows that any positive value of h will increase team scoring but it also shows that its impact is proportional to the magnitude of h, so it will be easier to visualize the patterns produced by varying the other parameters if h is at a maximum.

Design of the Computer Simulations. The simulations were not intended to be full simulations of real basketball players interacting on a court, and they were not designed to try to fit to specific sequences of hits and misses collected from basketball players. Instead they were designed to illustrate the impact of a nonzero value of *h* on scoring in interaction with the different values of the *b* (bias parameter) and *x* parameters. As in the Markov model, opposition players were not explicitly represented, instead it was assumed that the effects of the opposition were constants embodied in the bias and shooting ability parameters. (As discussed already, this assumption is consistent with Gilovich et al's data.)

The program simulated basketball shooting by selecting a player on each trial to take the next shot, and once such a allocation had been made that player was successful with a probability equal to its shooting ability parameter. To show the impact of nonzero values of the *h* parameter, a pair of simulations was run for each combination of the two free parameters (*b* and *x*). The *hot-hand simulation* assigned $h=1$. This value of *h* was instantiated by the following two-condition rule for allocating shots:

1) If a player has just hit its last shot, give that player the next shot and keep giving it to that player until a miss. (Because this also ensures that only one player can have hit its last shot, there is no need for a tie-breaking method.)

2) If neither player has hit its last shot, then the bias parameter is used to select a shooter by generating a random number higher or lower than that bias.

Thus the hot hand simulation setting of $h=1$ further simplified the simulation by always giving a player who hit the next shot, which would not be true for $h<1$. The paired *no-hot-hand* *simulations* assigned $h=0$, so shots were randomly allocated on the basis of the bias (b) parameter alone. In effect, Condition 2 was always applied regardless of whether the last shot hit or missed. Because all simulations were described by just two free parameters (bias and shooting ability), it was practical to explore the entire parameter space.

To test the effect of $h=1$ verse $h=0$ across the parameter space, pairs of simulations were run for all combinations of the b and x parameters. All parameter combinations for allocation values from 0.01 to 0.99 were run in increments of 0.01 (0.00 and 1.00 values would always give the shot to the same player, regardless of application or nonapplication of the hot-hand behavior), and all shooting ability values from 0.50 to 0.99 in increments of 0.01 (0.00 to 0.49 would simply repeat the other combinations). Thus the impact of the full range of parameter values on the effectiveness of the hot hand behavior can be illustrated.

Note that although the two simulations in a pair will be compared to each other, this is not intended to imply a competition between streaks and whatever the bias parameter represents (such as knowledge that one player has a higher shooting percentage) to find the one best allocation cue. The simulation pairs compare one that utilizes both streaks (i.e., $h=1$) and the bias parameter as allocation cues with one which only utilizes b (i.e., $h=0$), in order to determine what extra advantage is conferred by using streaks as an allocation cue as well as the bias parameter.

Results of the Simulations. For each parameter combination, ten million trials of the hot-hand simulation and ten million trials of the no-hot-hand simulation were run. The program then reported the total number of hits from all trials for each simulation. To determine the effect of the hot-hand behavior, for each parameter pair the number of hits produced by the no-hot-hand simulation were subtracted by the number of hits produced by the hot-hand simulation, then this difference was divided by the total number of trials to yield an *advantage* score representing extra hits per trial (range -1.0 to +1.0) which should have an expected value equal to the $h(b - b^2)(x - y)^2$ component of Equation 1:

$$\text{advantage} = \frac{\text{(hits for hot-hand simulation) - (hits for no-hot-hand simulation)}}{\text{total number of trials per simulation}}$$

Figure 2 presents a contour graph for the 4752 (48x99) pairs of simulations. This graph represents three dimensions of information: the bias parameter, the shooting ability parameter, and the advantage score in favor of the hot-hand simulation for that combination of parameters. Note that simulations using the 0.50 shooting ability parameter are excluded from Figure 2 because if there is no difference between players' shooting abilities, then there is nothing for the hot hand behavior (or the bias parameter) to exploit, therefore it is a 50% chance whether the hot-hand simulation comes out ahead. The numbered contours define boundaries in the distribution of advantage scores found for parameter pairs. For example, for every combination of parameters above the line labeled "0.2" the hot-hand simulation had at least 0.2 more hits per trial than the simulation without the hot-hand, whereas in the area between the contours labeled "0.2" and "0.1" advantage scores were between .2 and .1. The small solid black blob-like areas at the bottom of the graph labeled "0.0" indicate the few parameter pairs (.008 of all simulations) for which the hot-hand lost in this set of simulations. (To create these plots I used Sigma graph, which tries to smooth contours, which can result in odd shapes when there is nothing systematic.)

Of the 4752 pairs of simulations run, only 43 yielded negative advantage scores, which is why virtually the entire area of Figure 2 represents advantages to the hot hand simulations, as Equation 1 predicted. Thus using streaks as an allocation cue as well as the bias parameter, was almost always superior to using the bias parameter alone as an allocation cue. When the hot-hand simulation did come out behind, the negative result was very small with only -0.0018 hits per trial being the most negative advantage score recorded. Not surprisingly, the greatest advantage for the hot-hand simulation occurred when shooting ability was high and the bias parameter low, as the bias parameter's effectiveness is a function of how strongly it tends to allocate shots to players with greater shooting abilities. The poorer it is at this, the greater the relative effectiveness of the hot hand rule in increasing the allocation to the player with the higher true likelihood of success. (For example, a bias parameter based on the career shooting percentage of an injured player may be a relatively poor allocation cue.)

The few parameter pairs that did show a loss by the hot-hand in these runs were in the regions most expected. The hot-hand rule, like any allocation strategy, has its least effect when the two players are the least different from each other, that is, when the two shooting abilities are both very close to .50. There was also a slight tendency for more losses when the bias parameter was very close to 0.0 or 1.0. When this parameter is close to these extremes, then the same player will almost always be allocated the shot; thus the impact of any other allocation strategy will be very small. However, it appeared that even under the least favorable conditions, the hot-hand behavior did no harm, although it could confer too small of an advantage to always generate a win.

One question that may be raised is whether it might be critical that one player had a shooting ability below .50 and the other above .50. To check this, a set of simulations were run which were identical to the above set, but the sum of the two shooting abilities for the players equaled 0.50, thus the higher shooting ability parameter was varied from 0.25 to 0.49. Very similar results were obtained with the hot hand only recording occasional loses when the two shooting abilities were very close. The same result was found for simulations in which the sum of the two shooting abilities was 1.5 so that both were greater than .50.

By showing an advantage for $h=1$ over $h=0$ across virtually the entire parameter space (i.e., all combinations of all values of the bias and shooting ability parameters), the simulations showed that the specific values of the shooting ability and bias free parameters were not critical. The hot-hand behavior conferred an advantage even under conditions that accommodate other heuristics being employed in addition to the hot hand. For example, probability matching (Estes, 1954) is a strategy that has been found to be often followed by people and animals. However, when the allocation and shooting abilities are equal (which could be represented by a diagonal line from the bottom-left of Figure 2 to top-right) the hot-hand simulations almost always won. It would also seem that for such simulations, just giving the shot almost always to the player with the highest shooting ability may eliminate the influence of following the hot-hand rule. Yet even when the bias parameter was set at extreme values, such as giving the ball to one player 99% of the time, the addition of the hot-hand rule still led to more shots being hit. This is consistent with Equation 1.

Thus the simulations were consistent with the analysis of the Markov model. If weight is given to bias to follow streaks, then scoring should increase, everything else being equal.

Under what conditions is following streaks adaptive?

The variables and parameters in Equation 1 need make no reference to basketball, thus the analysis shows that following streaks might be an advantage in many domains. Basketball is not the only situation in which people are faced with repeated opportunities to choose between the same options. As already pointed out, this is common in sports, but also in investment decisions. Many types of decision making are sequences, from asking different agencies to send job candidates to selecting a restaurant. Animals foraging are faced with sequences, but so are people seeking the cheapest groceries. Psychology experiments often present sequences of the same decisions, yet they are rarely examined in that way (an exception is Soetens, 1998). Understanding under what conditions streaks are valid allocation cues will provide a starting point for understanding how people use sequential information, and it is a necessary precursor to empirical work on when streak information will be utilized.

Although following streaks is adaptive under the conditions that Gilovich, et al.’s (1985) data show apply during basketball, this behavior will not be adaptive under all conditions. Based on Gilovich et al.'s data, four important conditions can be identified that to apply to basketball: 1) for all options, the success of an option was independent of previous successes; 2) different options varied in their probabilities of success; 3) probabilities of success were constant; 4) no other entity reacted to the streak such as to have a discernable impact on either the success or allocation of shots (although Equation 1 predicts that such a behavior would be adaptive for defenses, a sort of reverse hot hand behavior). In order to understand the general condition under which following streaks should be successful, I will consider the implications of relaxing each of these conditions.

Condition 1: Independence

If successes for an option were positively dependent, as the hot hand belief implies, then using streaks as an allocation cue should be adaptive. This accords with many common beliefs such as "success breeds success." Equation 1 shows that even when successes are independent then following streaks is adaptive (given the other conditions that apply to basketball), but what if successes are negatively dependent, surely then following streaks would be maladaptive? This is not necessarily so.

The Markov model in Figure 1 can be modified by adding a new parameter d (range: 0.0 to 1.0) which could represent a reduction in the probability of an option being successful after a success. For example, a strong defense may be applied against players who have hit their last shot. This parameter would decrease the probability of Player X hitting a shot after a hit. Instead of *p(X hit | X hit last) = x* now *p(X hit | X hit last) = x(1-d)*, similarly *p(Y hit | Y hit last) = y(1-d)*. This parameter should also increase the probability of the success of other options (for example, because pressure on one player may decrease pressure on the others), such that *p(X hit | Y hit last) = x + d(1-x)* and *p(Y hit | X hit last) = y + d(1-y).* When *d=0* there is no negative correlation between successive shots, but when *d=1* the option that was successful on its last opportunity is never successful the next time it is selected. All positive values of *d* should produce a negative correlation for the success of successive shots. Note that there is no reason to assume that the parameter that decreases the chance of a player hitting after a successful shot should be identical to the parameter increasing the probability of the other player hitting the shot. In the following analysis I have assumed that these parameters are identical in order to make the equations easier to follow. I have done the same analysis assuming two different parameters, and found similar conclusions.

If the *d* parameter is added to Figure 1 using the conditional probabilities already specified, then the following augmentation of Equation 1 represents the expected outcome (a full derivation of this equation is available from the author on request):

$$(2)\qquad E(\text{hits after two shots}) = 2(b(x - y) + y) + h(b - b^2)(x - y)^2$$
$$+ d((b - b^2)(x - 2xy + y) - b^2 (x^2 + y^2) - y^2(2b-1))$$
$$- hd(b - b^2)((x - y)^2 + x + y)$$

In order to determine for what values of *d* does following streaks help performance, the critical inequality is:

(3) $h(b - b^2)(x - y)^2 - hd(b - b^2)((x - y)^2 + x + y) > 0$

$\Rightarrow (1-d)(x - y)^2 > d(x + y)$

When the inequality in Equation 3 will be true depends on the particular parameter values for *x*, *y* and *d*. Thus even when successes for sequential events are negatively correlated, streaks of success may still be a valid allocation cue.

On one level, the finding that belief in the hot hand could often help even in the face of negative dependencies between events seems very surprising. However, given that Equation 1 showed that belief in the hot hand was beneficial, it should not be surprising that not all negative dependencies would be big enough to overcome that advantage. However maybe anything but a tiny value of *d* will produce a negative outcome if *h* is not zero. Substituting some values for *x*, *y* and *b* can test this idea. Let us assume $x = .75$ (probability of Option X being successful) and $y = .25$ (probability of Option Y being successful), which are not very realistic parameter values for basketball but possible for other types of streaks. With these parameters, the two sides of Equation 3 are equal when $d = .20$. Thus even a 20% decrease in the probability of a hit following a hit and 20% increase in the probability of the other option producing a hit (which should produce a strong negative correlation) could still result in no detrimental effects on success from following streaks.

This analysis has an interesting implication. If belief in positive dependencies encourages the behavior of following streaks, then Equation 1 shows that this will be beneficial when events are independent. However Equation 3 shows that this belief may still be beneficial even when it is exactly the opposite of reality, that is, when events are negatively dependent. Equation 3 suggests that such a fallacy is most likely to be beneficial when the differences between probabilities of success are relatively high compared to the magnitude of the negative dependency. The strength of the bias to follow streaks (*h*) is not a critical factor for determining when this behavior is adaptive.

Condition 2: Equal probabilities of success

In order for any allocation cue to be beneficial, there has to be some exploitable difference between the options to which the next opportunity can be allocated. Clearly, Equation 1 predicts that when all options have the same probabilities of success (and if independence holds) then there is no advantage to following streaks. However, if people are sensitive at some level to the implications of the analysis embodied in Equation 1, then they should be less likely to follow streaks when they think there is no difference between the options. Sequences that fulfill these conditions fit to most definitions of *randomness*.

Defining randomness has been problematic (Nickerson, 2002), but its definitions often focus on three features of a sequence considered random (see Rapoport & Budescu, 1992; Wagenaar, 1991): 1) a fixed set of alternatives; 2), a selection procedure that does not utilize previous outcomes (i.e., independence of events); and 3), a selection procedure with no preference for any of the outcomes (i.e., equiprobability of events). There is no advantage in favoring one event over another when someone is faced with a sequence of events with these characteristics. However if people think of the process as "nonrandom" then it implies a violation of at least one of these conditions, and thus it is possible that following streaks may be advantageous.

The implication that streaks should help when the options have unequal values but not when they are equal, presents a way to empirically deal an unresolved issue about the hot hand as a belief about streaks: why not apply the gambler's fallacy instead? Faced with a streak of events and the necessity to make a choice, people may make one of three possible inductions: 1) that the streak is irrelevant, 2) that the streak will continue, 3) that the streak will stop. When faced with a forced choice, if people generally accepted the first of these inductions then they should predict the next event with a probability equal to its base-rate (e.g., 50% for a fair coin flip). However, what is often observed is a bias towards one of the other two inductions, even when the events are independent. The hot hand behavior represents the second induction (that the streak will continue), whereas the third represents the gambler's fallacy, a tendency to believe that a streak of events is likely to end. Laplace (1814/1951) first wrote about this phenomenon, and its existence has been well documented (Tune, 1964).

Tversky and Kahneman (1971) explained belief in the gambler's fallacy as due to the representativness heuristic leading to a belief in a *law of small numbers*. In order for a sequence of events to be considered representative, people think that every segment of a random sequence should reflect the true proportion. Thus a streak of one type of event must quickly end and be “evened out” by the other events. Gilovich et al. (1985) argued that belief in the hot hand is also due to belief in the law of small numbers. A belief that things should "even out" will be challenged by a long streak, therefore basketball players may reconcile the apparently unusual streak and their belief in the law of small numbers by assuming that the events are dependent. This may succeed as a description of the phenomena, but provides no way of predicting when people will favor which induction.

Falk and Konold (1997) point out that, in general, representativeness offers a convincing account of what participants do when judging random sequences, but its predictive power is weak. Similarly Gigerenzer (2000, p. 290-291) pointed out that it is problematic to explain the opposite phenomena with the same principle, yet this is what has been done with the gambler’s fallacy (i.e., the streak should stop) and the hot hand (i.e., the streak should continue). A step towards understanding how people use streak information is trying to understand what distinguishes situations in which people tend to think a streak will continue, and when they tend to think it will stop. The law of small numbers cannot explain this.

Equation 1 seems to have a clear implication for behavior. When the probabilities of success for different options are unequal then it is adaptive to follow streaks. However when the probabilities are equal it does not matter whether a streak is followed or not. If following streaks is more likely to be applied in situations in which it is adaptive, then people should be more likely to follow streaks when they think the probabilities of events are unequal than when they think they are equal, that is, when they think events are random.

Burns and Corpus (in press) tested this by describing a streak of events to participants but varying how random the process generating events were judged to be. Three scenarios (two people competing for the highest sales each week, free-throw shots, roulette wheel) were presented which varied from each other in terms of how random they were judged to be (as shown by ratings on a 7 point scale from "completely random" to "completely nonrandom"), and asked to predict the next event. Thus it could be determined for each scenario how likely participants were to predict the streak to continue or end. Participants rated the result of a competition for sales as least random, but were most likely to predict the streak the salespersons generated to continue. In contrast, they rated the result of roulette wheel spins as most random, but were least likely to predict the streak it generated to continue. Thus the empirical implications of Equation 1 provided some explanation for why the same streak can be treated differently. Burns and Corpus' results imply that people are more likely to follows streaks in situations in which they perceive that options are random, but instead carry on streaks when they perceive that options may differ in their probabilities of success. In turn, these are exactly the conditions under which following streaks is most adaptive, implying that people are sensitive in some way to the implications of the analysis of the hot hand phenomenon presented here.

Condition 3: Constant probabilities of success

Equation 1 shows that following streaks yields an advantage in general, but it assumes a completely stable environment. This is why it is at a maximum when *b*=1.0 (i.e., the bias parameter is set to give the same player every shot). When *b*=1.0, the parameter value of *h* is irrelevant. This points to a limitation of Equation 1 in that it does not model the possible variability in the probability of the success of options. Gilovich et al. (1985) found no evidence of variability in shooting percentages, but it is unclear whether it is inherent to basketball that players never vary in their ability to hit shots.

Perfect stability in the true likelihoods of success of all options is probably rare, although exceptions would be casinos and psychology experiments in which probabilities are fixed. Despite Gilovich, et al.'s (1985) results showing stability in shooting percentages, they are probably not characteristic of professional basketball under all conditions. For example, imagine if a NBA team actually tried to implement allocating 100% of all shots to their one best shooter in an actual game. Once the opposition realized that a team was giving 100% of its shots to one player, then that player would probably be surrounded by five opponents every time he received the ball. One of two things would then happen: 1) either that player would cease being given 100% of the shots (i.e., set $b < 1.0$), which would create a situation in which the hot hand behavior is effective; 2) that player's shooting percentage would decrease. The latter case violates Gilovich, et al.'s data implying that shooting percentages are constants. Thus the stability that Gilovich, et al. found may be a product of the low *b* parameters (i.e., bias to give the shot to a player) that characterize real-life high-level basketball. In their data, the player who took the highest percentage of all the shots was Julius Erving who took 23.3% (884/3800). Daryl Dawkins had the best shooting percentage, but apparently was a player who took most of his shots from very close to the basket, which reinforces the fact that factors other than shooting percentage and streaks are used as allocation cues. However, even if Erving's shots were conditionalized on where they were taken from and it was taken into account that he was not on the basketball court for every minute of every game, this seems to indicate that NBA teams deviate a long way from what would be optimal if shooting percentages were stable under all conditions (i.e., set b=1 for the player with the best percentage). For all other allocation strategies, Equation 1 shows that a nonzero h parameter should be beneficial. The likelihood that stability in shooting percentages would break down if b parameters were set too high, provides a simple explanation for why the pure strategy of giving the ball every time to the single best shooter is never likely to be observed in an NBA game. In this case though the variability would be likely to be systematic (i.e., a negative correlation between b and shots hit), but what about the more general case in which there may be variability in true probabilities of success which is not related to any of the other parameters in a predictable way?

A general version of the model in Figure 1 could relax the assumption that probabilities of success are stable, though how best to do so would require making further assumptions. However, most ways of modeling instability of probabilities would tend to imply that following streaks would be even more advantageous than when probabilities are stable because streaks carry information, as Table 1 shows.

If streaks carry information, then that information is most useful when there is an absence of other information or when that information in unreliable. For example, players in a pick-up game of basketball who do not know each other may gain a greater advantage for their team by using streaks as an allocation cue than would NBA players who know their teammates well. For sequences of events in which probabilities of success change over time, following streaks has the advantage that it exploits something (change in the probability of a streak) that occurs immediately if there is a change in the probability of the event that might be part of a streak. In contrast, behavior based on frequencies (such as shooting percentage) or most learning mechanisms will take longer to react to changes in the underlying probabilities, because they are based on accumulated history. So a belief that streaks will tend to be continued may be a useful heuristic in many situations. For most sequences of events, the conditions that may apply to NBA basketball - perfect knowledge of the base-rate and no fluctuations - are likely to be invalid, and in this sense the NBA may be the worst place to look for advantages from following streaks. Models of foraging behavior such as McNamara and Houston (1985) have shown that the more unstable the environment, the greater weight should be given to recent information. Similarly Lovett (1998) found that to accurately model probability learning when probabilities varied it was critical to incorporate decay into her model. In effect decay gives greater weight to recent experience. Thus following streaks may be more effective behavior in unstable environments than in the relatively stable NBA in which reliable information is available.

Even if the environment is stable, it may not necessarily be a valid induction that it will stay that way. Thus one reason why people may fail to maximize their outcomes in probability learning experiments (e.g., Estes, 1954) by always choosing the better option, may be that they are uncertain that it will always be the better option. In an unstable world, failure to ever sample other possibilities may be nonadaptive, which may explain why pure strategies of picking the same option every time (i.e., setting $b=1$ and rendering the h parameter irrelevant) are rarely observed. Even in a stable world in which the same option is always the best, following streaks may provide an advantage, given that people will rarely apply the optimal strategy of always choosing the one best option. In such a situation, following streaks will result in the best option being chosen more often than it would be if streaks were not followed. Thus utilizing streaks as an allocation cue may be an adaptive heuristic for reducing a performance deficit caused by the consistent use of a nonoptimal strategy due to uncertainty.

If the more variable the environment, the greater the benefit from following streaks, then people should be more likely to follow streaks in sequences they see as generated by unstable processes than by stable processes. Burns (2003) provided a preliminary empirical test of this proposition by providing participants with information indicating that two companies had experienced the identical streak in their stock price. However one company was big and old, whereas the other was small and new. Almost all participants indicated that they though the small company would have a less stable stock price than the big company. As predicted, whether the streaks for both were an increase or a decrease in stock prices, participants were more likely to predict that the small company would continue the streak.

Condition 4: Intervention against streaks

Probabilities may be unstable, not just because there is variability in the environment, but because some agent works actively to alter them. Opponents in basketball are a possible example of such active agents. As discussed above, the stability that Gilovich et al. (1985) found implies that in their sample pressure from opponents did not affect shooting success, at least to the extent to which it was applied by defenders in response to streaks. However, this may only be true when players avoid strategies that place too great a reliance on a single cue. Utilizing a mixed-strategy does this, just as game theory would predict for a game in which there is no pure strategy that is guaranteed to work. As discussed already, at the levels of the b and h parameters that appear to be in effect for NBA players, they appear not to be high enough to evoke a contra-strategy that affects shooting percentages. Otherwise there should have been have been significant negative dependencies found in Gilovich et al.'s (1985) data. Thus although Equation 1 shows that players' utilization of the hot hand and biases based on shooting percentage as allocation cues must be helping them, it cannot be seen as an argument that it is rational for NBA players to raise their utilization of either of these cues to their highest levels. A contra-strategy triggered by higher values may make hitting basketball shots no longer independent and therefore invalidate Gilovich et al's data and Equation 1.

However if a bias to following streaks is correlated with effective action by an opposing entity, then it may not be beneficial. For example, in basketball defenders may believe in the hot hand such that they defend so strongly against players who hit their last shots that they introduce a negative dependency between successive shots. Although Gilovich et al's (1985) data show no evidence of this in NBA basketball, it is a general condition under which following streaks may not be adaptive.

Summary: Conditions under which following streaks should be effective

From the above discussion can be summarized the general conditions under which following streaks would be adaptive for choices between a fixed set of mutually exclusive options[2] Following streaks should be adaptive when: 1) events are positively correlated, independent, and sometimes when they are negatively correlated (depending on the extent of the advantage one option has over another); 2) the probabilities of success of different options are not equal, and the greater the inequality the greater the advantage; 3) there is variability in the probability of success, or when the environment is stable but the organism does not utilize the optimal strategy because of uncertainty that this stability will continue. Following streaks will *not* be adaptive under two specific conditions: 1) when the successive options are negatively correlated over time, as long as any difference in the probabilities of success of different options is not too high; 2) when following streaks encourages effective negative action on the part of some other entity that overcomes any advantage. As can be seen, the conditions under which following streaks is adaptive are likely to apply to greater range of situations than the conditions under which doing so will be harmful, unless the world is structured in a very unusual way. Thus following streaks may be a generally applicable adaptive heuristic.

An empirical link between the hot hand belief and the hot hand behavior

If the behavior of following streaks is often adaptive this raises the question of what connection it may have to the common belief in dependencies between successive events. Gilovich et al. (1985) appeared to assume that the hot hand belief and the hot hand behavior are connected, yet there is no empirical evidence that basketball players or fans see a connection. Furthermore Gilovich et al. suggested that the hot hand belief is due to fans/players observing streaks that violate their belief in the law of small numbers. My analysis suggests an alternative explanation: that the hot hand belief may be at least partly due to the adaptiveness of the hot hand behavior. My analysis suggests that implementing the hot hand behavior should improve the performance of a team, and therefore players may have learned to apply the behavior as a result of playing a large amount of basketball. The hot hand belief may be the way that players explain to themselves why they follow the hot hand behavior.

The argument that the hot hand belief exists to support the hot hand behavior suggests that there should be a connection in the mind of players between the belief and the behavior, and that this connection should be stronger for players with more experience of basketball. To test this, I presented participants who varied in terms of basketball experience with the hot hand belief question (Statement 1) and the hot hand behavior question (Statement 2) that Gilovich et al. (1985) presented to a sample of 100 basketball fans. I expected that there should be a strong association between participants' answers to the two questions. In addition, participants were asked to explain explicitly why they endorsed the hot hand behavior, and what implications the hot hand belief had.

Gilovich et al. (1985) put forward the view that the belief in the hot hand by experienced basketball watchers or players was due to their experience exposing them to streaks that they interpreted as violating the law of small numbers. The alternative view presented here is that the belief may arise amongst basketball players because they experience the success of the hot hand behavior and the hot hand belief is a way of explaining to themselves the phenomenon, and thus making it more likely that it will be maintained. Thus the hot hand belief could be due to adaptive learning. (The argument that following streaks may be effective across a wide range of conditions suggests that even novices may follow streaks in basketball, but such a tendency may be reinforced by experience.) Both the law of small numbers view and the adaptive learning view would predict that players with more experience should be more likely to endorse the hot hand belief.

The adaptive learning view also predicts that players with more experience should be more likely to endorse the hot hand behavior. The law of small numbers may also lead to this prediction given that the behavior would seem a natural consequence of the belief. Logically, any player who thinks that current circumstances make some player more likely to score should favor giving that player the ball. Where the two views may diverge is in terms of the link between the hot hand belief and the hot hand behavior. The adaptive learning view predicts that the association between the belief and the behavior should become stronger with experience. Thus the more experienced players are the more likely they should be to explicitly explain their endorsement of the behavior as due to the belief, and to claim that endorsement of the belief implies the behavior should be followed. The law of small numbers would seem to simply imply greater rates of endorsement of the hot hand belief and the behavior with greater experience, but not to predict a stronger linkage between them amongst players with more experience.

Method

Participants

A total of 1362 members of the Michigan State University subject pool participated for partial course credit. The sample was not recruited on the basis of experience with basketball, therefore a large sample was necessary in order to obtain sufficient participants with high levels of basketball skill. Gilovich et al. (1985) only gave their questions to basketball fans, but they did not specify what defined someone as a fan, or where this sample was drawn from. Therefore direct comparisons can not be made to their data.

Materials and Procedure

Participants received two separate sheets of paper, one dealing with the hot hand belief and one with the hot hand behavior. The sheets were given to participants in a random order and separated by unrelated reasoning tasks that required at least 15 minutes to complete. Each sheet asked participants about either the hot hand belief or the behavior, in particular did they endorse it or not. Participants were then asked to explain the implication of their answer by selecting from multiple options, including an open-ended possibility. Thus they were not forced to give an answer they may not believe simply because it represented the best of a poor set of choices.

The *hot hand belief* sheet first posed a question similar to the one posed by Gilovich et al. (1985) to assess this belief, "Does a college/professional basketball player have a better chance of making a shot after having just made his/her last two or three shots than he/she does after having missed his/her last two or three shots?" Participants responded by circling either "Yes" or "No" then they were asked "What implications for college/professional basketball does your answer have for who to give the next shot to?" Participants were presented with four options and asked to circle one:

A. It is important to pass the ball to someone who has just made several (two, three, or four) shots in a row.

B. It is important NOT to pass the ball to someone who has just made several (two, three, or four) shots in a row.

C. My answer has no implications for who to pass the ball to.

D. Other (Please explain):____

They were then asked to write down an explanation for their answer.

The *hot hand behavior* sheet first posed a similar question to another one posed by Gilovich et al. (1985) to assess belief in the hot hand, "In college/professional basketball is it important to pass the ball to someone who has just made several (two, three, or four) shots in a row?" Participants were then asked to "Please indicate which of the following reasons led you to this answer (please circle MORE than one if you think there are multiple reasons, but please indicate which reason was most important as well)." However they were presented with a different set of options depending on how they answered the hot hand behavior question. If they answered "yes," the three options were:

A. A player has a better chance of making a shot after having just made his/her last two or three shots than he/she does after having missed his/her last two or three shots.

B. A player who has made several (two, three, or four) shots in a row probably has a better career shooting percentage (i.e., is consistently better) than the other players.

C. Other. (Please explain

If they answered "no" the three options were:

A. It doesn't matter who hits the last few shots when deciding who should take the next shot.

B. A player who has made several (two, three, or four) shots in a row is due for a miss.

C. Other. (Please explain)

Before participants were presented with either sheet, they indicated their experience with playing basketball by specifying which of the following applied to them (they could specify more than one): I played on a college team; I played on my high school varsity team; I played on my high school junior varsity team; I played intramural basketball; I play a lot of pickup games; I play some pickup games; I play occasional pickup games; I have played very little basketball; I have never played basketball. As a way of providing some validation of this measure of experience they were asked to "Please try to roughly estimate about how many basketball games (official and pick-up games) you think you have played in your life?"

Results

Table 2 shows that there was a strong association between how participants answered the basic hot hand belief and hot hand behavior questions, $X^2(1) = 376.0$, $p < .001$. Seventy-seven percent of participants gave the same response to the two questions.

To look at the relationship between expertise and how participants responded to these questions, I classified participants into one of three skill categories based on their answer to the experience question. The 580 participants who responded either "I have played very little basketball" or "I have never played basketball" were classified as *low skill*. The 272 participants who indicated "I played on a college team," (7 participants), "I played on my high school varsity team" (196 participants), or "I play a lot of pickup games" (69 participants) were classified as *high skill*. The remaining 510 participants were classified as *medium skill*. Participants' answers to the games played question were highly skewed, and how reliable their answers were was impossible to gauge, but their answers provided some validation to the skill categories. To deal with the skewed distribution of answers I arbitrarily assigned to all answers above 300 games the value 300. This affected only 9% of the participants. The mean number of games played by the low skill group was 10.4 (SD = 17.4), which was less than for the medium skill group (M = 85.1, SD = 84.1), which in turn was less than for the high skill group (M = 194.5, SD = 101.9). The skill effect was statistically significant, $F(2,1338) = 532.9$, $p < .001$.

Table 3 presents participants' answers to the hot hand belief and hot hand behavior questions as a function of their skill level. The hot hand belief was more common as a function of skill level, $X^2(2) = 23.1$, $p < .001$. The comparisons of low to medium skill, $X^2(1) = 5.72$, $p = .017$, and medium to high skill, $X^2(1) = 7.72$, $p = .005$, were statistically significant. Table 3 also shows that the endorsement of the hot hand behavior was also stronger as a function of skill level, $X^2(2) = 49.3$, $p < .001$. Both the comparison of low to medium skill, $X^2(1) = 18.0$, $p < .001$, and medium to high skill, $X^2(1) = 10.9$, $p < .001$, were statistically significant. The percentage of participants agreeing with both questions also increased with skill level (Low 38%, Medium 51%, High 64%), $X^2(2) = 50.2$, $p < .001$,

To examine what participants thought were the implications of the hot hand belief, I examined the reasons given by the 848 participants who answered "yes" to the hot hand belief question. Answer A represented the *pass* response, Answer C the *no implication* response, and

Answer B represented the *don't pass* response. The 70 participants who gave the answer D, representing "other" were classified into one of the other three responses on the basis of their explanation (this classification was done blind to their responses on the other sheet). Table 4 shows the relationship between participants' skill level and what they saw as the implication of the hot hand belief. Overall, there was a relationship between skill level and implication, $X^2(4) = 26.6$, $p < .001$. Comparing just those participants who indicated that the belief implies passing the ball to the player on a streak versus those answering differently, there was a statistically significant difference between participants with low and medium skill levels, $X^2(1) = 5.72$, $p = .017$, and between participants with medium and high skill levels, $X^2(1) = 9.25$, $p = .002$. Overall, for participants indicating that shots have a positive dependency, those with higher skill levels were more likely to indicate that this belief had the implication that that player should be passed the ball.

When the 782 participants who indicated that they thought it was important to pass the ball to a player on a streak were asked to explain why, most answered with either "A," which represented the *hot hand* explanation, or "B" which represented the *better player* response. All but 5 out of 67 participants who answered "C" (other) could be classified into another category on the basis on their written explanation, but some participants indicated that they thought both the hot hand and better player explanations were correct (participants were free to indicate more than one explanation, though they were asked to indicate which they thought was more important if they did so). Table 5 show that the *hot hand* explanation was more common as a function of skill level, $X^2(4) = 42.2$ $p < .001$. Dichotomizing between those indicating that the hot hand belief alone was the best explanation for why they endorsed the hot hand behavior and any other response, there were significant differences between those participants with low and medium skill, $X^2(1) = 10.3$, $p = .001$, and those with medium and high skill, $X^2(1) = 11.0$, $p = .001$.

Discussion

The study showed that there was a strong relationship between belief in the hot hand and thinking that players experiencing a streak should be given the shot (i.e., support for the hot hand behavior). In addition the more skilled were the participants, not only were they more likely to endorse the hot hand belief and the hot hand behavior, but the more likely they were to express a link between them.

Both the law of small numbers and the adaptive learning views of the hot hand belief would seem to predict that the more skill and experience participants had with basketball, the more likely they should be to hold the hot hand belief. Both may also predict that the higher a participant's skill level, the more likely they were to endorse the hot hand behavior of giving players experiencing a streak the ball. However, the law of small numbers does not seem to predict that there should be a stronger relationship between the hot hand belief and the hot hand behavior as a function of skill. Why should not any participant thinking that a certain player has a greater chance of scoring also favor giving that player the ball, irrespective of their skill level?

Of course one criticism of the law of small numbers has been how hard it is to base predictions on it, so perhaps there is some way it can be used to explain these results. Thus I will not claim anything stronger than that these data provide some evidence consistent with the alternative approach to explaining the origin of the hot hand belief among basketball players. The adaptive learning approach suggests that the belief may arise not just because players have seen streaks that appear to contradict their assumptions about randomness, but because they have experienced the success of following streaks and the hot hand belief is the best explanation they have for that. However, the data here is not definitive evidence for this view, so perhaps some alternative explanation is possible. For example, the association between the hot hand belief and hot hand behavior may be due to younger players hearing it from older players, although this begs the question of why the association arises in the first place.

The adaptive learning view suggests that the people who have played more basketball, and hence had more exposure, are more likely to utilize the hot hand behavior because they hold the hot hand belief. We know that people are sensitive to learning covariations (Hasher & Zacks, 1984), so if a bias to give the next shot to a player who has hit his or her last shot tends to increase a team's scoring, then players are likely to learn this. This does not necessarily mean that their explanations for why they should give the ball to a player who is hitting shots will be accurate. Even when given the option of supporting their endorsement of the behavior of allocating shots to players experiencing streaks with what my formal analysis shows is the *correct* belief (that such players are likely to be better shooters) they instead justify their endorsement with the *incorrect* belief (that shots are conditionally dependent). Holders of this incorrect belief are more likely to endorse the correct behavior. The work reviewed by Nisbett and Wilson (1977) suggests that people may often make decisions without conscious awareness of the true reasons why they made those decisions, and then they may make up plausible sounding reasons for their behavior. Often people look for causal explanations for events, and that one shot influences the next may well be the most plausible sounding causal mechanism. So the erroneous beliefs fans and players have about streaks by individual players may simply be their best attempt to explain a behavior they have learned is adaptive. Thus belief in streaks for individuals may be a misunderstanding by players of the reasons for an accurate perception of the hot-hand as it applies to allocation of shots among members of a team, rather than the misperception of sequences which appears to be the basis of the gambler's fallacy.

Although my rough calculation of the advantage the hot hand behavior may provide to NBA teams was quite small, NBA players have had a huge amount of experience before they become professional. The conditions under which players grow up playing basketball, as opposed to those present in the NBA, are likely to be much closer to the general conditions (already discussed) that should increase the adaptiveness of following streaks. In particular, on the playground there may be a lack of base-rate information about other players' shooting and young players may experience change and variability in their own and their teammates true shooting ability.

Asking the right question: Beliefs verse behaviors in decision making

This paper has tried to make in great detail a point that may now appear obvious, as it is based on the straightforward mathematical argument that streaks must be predictive of the true likelihoods of independent events. Why then has the hot hand been seen only as a fallacy? One reason may be that the previous analysis of the hot hand has been embedded in a particular approach to decision making that focuses on how normative are people's beliefs. Stanovich (1999) classifies different approaches to human rationality in terms of different relationships between three types of models of reasoning: the *descriptive*, what people actually do; the *prescriptive*, what people should do; and the *normative*, what analytically would be the correct choice. Stanovich points out that although some philosophers may defend the proposition that all three models are equivalent, few psychologists would due to the strong empirical evidence that the descriptive and normative models often diverge. However, the approach pioneered by Kahneman and Tversky, which may be referred to as the *heuristics and biases* approach (see Gilovich & Griffin, 2002), has assumed that the normative and prescriptive models are very close. In contrast, the adaptive approaches of both Gigerenzer and John Anderson have argued that the normative and prescriptive may diverge, and furthermore they argue that when they diverge then human behavior tends to be closer to the prescriptive. Similarly, researchers in the field of *naturalistic decision making*, also reject the assumption that models of human decision making must start with normative choice models (Lipshitz, Klein, Orasanu, & Salas, 2001a). Lipshitz, Klein, Orasanu, and Salas (2001b) characterize the response to their position by some researchers who see the normative model as prescriptive as a Kruschevian 'we will bury you.' For example, Bazerman (2001) seemed to argue that standards derived other than from the normative model were fine, as long as they did not clash with the normative model. Overall, the assumption that the normative *is* the prescriptive has been the dominant approach as Hastie (1991) pointed out in opening his review of judgement and decision making with 'Will the field ever escape the oppressive yoke of normative "rational" models?' (p. 137). Despite extensive work (e.g., Kahneman, Slovic, & Tversky, 1982) refuting them as models of human behavior, rational models such as expected utility were still being used as the ideal. Thus the rejection of the rational choice models has only been partial.

In the robust exchange between Kahneman and Tversky (1996) and Gigerenzer (1996), a critical area of dispute was Gigerenzer's claims regarding what constitutes an appropriate way to judge decision making. However this argument over what is the appropriate standard against which to compare decision making seems to partly hinge on another distinction: should a decision be assessed in terms of the validity of the beliefs on which it is based, or on the adaptiveness of the behavior that results?

Tversky and Kahneman (1974, p. 1124) opened their seminal article with "Many decisions are based on beliefs concerning the likelihood of uncertain events… What determines such beliefs?" By focusing on beliefs the heuristics and biases approach to research on decision making was constructed with the main aim of identifying the beliefs people hold, often by making inferences from apparently incorrect behavior. Incorrect behavior seemed almost a necessity for such an approach as otherwise the beliefs would be hard to infer, just as visual illusion may tell us more about vision than stimuli people correctly identify. In contrast, the adaptive approaches of Gigerenzer (2000) and Anderson (1990), as well as the naturalistic decision making approaches (Lipshitz, et al., 2001a), have focused on assessing whether behavior achieves goals. Thus the distinction between assessing behavior or beliefs can be seen as a major difference between the heuristic and biases approach and the alternative approaches.

Interestingly, there have been recent attempts by Gilovich and Griffin (2002) and Samuels, Stich and Bishop (2002) to argue that the differences between the heuristics and biases approach and the adaptive approach are minimal. Gilovich and Griffin point out that both approaches suggest that people use heuristics that should usually succeed but at least sometimes will not, and further that they do not differ in terms of adaptiveness because "the heuristics and biases approach … *is* an evolutionary account" (Gilovich & Griffin, p.10, italics in original). However, if the evolutionary origin of heuristics is to be emphasized, then this suggests that behavior should be the criterion for assessing them. Evolution can only be driven by the effectiveness of behavior in an environment. Beliefs do not help an organism pass along their genes except to the extent that they have an impact on behavior. Yet Gilovich and Griffin (2002) state:

> There is, however, one version of this critique [that "we cannot be that dumb" by evolutionary psychologists] to which researchers in the heuristics and biases tradition must plead 'no contest' or even 'guilty.' This is the criticism that studies in this tradition have paid scant attention to assessing the overall ecological validity of heuristic processes. (p. 8)

By ecological validity, it seems they mean whether the behavior generated by the heuristic is successful (this is how Anderson, 1990, used the term). Gilovich and Griffin argue that such an assessment would be too large a task, so "the focus has been on identifying the cues people use, not on evaluating the overall value of those cues." (p. 8) Gilovich and Griffin appear to view themselves as speaking in general for researchers taking the heuristics and biases approach, and they may well be right given that this statement seems entirely consist with Tversky and Kahneman's (1974) aim of identifying the beliefs people hold. Gilovich and Griffin point out that the heuristics and biases approach and the adaptive approaches "share a set of assumptions: the ecological validates are probably high, the heuristics are generally useful, but common and profoundly important exceptions are to be found." (pp. 8-9) This statement is correct to some extent, but where I believe that Gilovich and Griffin fail to appreciate a core part of the critique of the heuristics and biases approach is that the ecological validity and usefulness of an heuristic should never be *assumed*, instead these are the factors by which heuristics should be assessed.

Correct beliefs and adaptive behavior should often co-occur. Often a rational belief will lead to adaptive behavior, whereas an irrational belief may often result in nonadaptive behavior. This tends to obscure the distinction between studying beliefs and behaviors. However, a danger arises when this link is assumed as there is no fundamental reason why it must exist, as Anderson (1990) pointed out. The hot hand phenomenon illustrates this because it appears to represent a situation in which the validity of a belief can be separately assessed from the adaptiveness of the behavior it is associated with. Furthermore, analysis of the hot hand shows that assessing behavior or belief is not just a matter of taste but can lead to opposite conclusions. Following the supposed behavioral implication of the logically correct belief (i.e., ignore streaks as an allocation cue) would result in poorer performance than following the behavioral implication of the logically incorrect belief (i.e., utilize streaks as an allocation cue). Thus a valid belief leads to nonadaptive behavior, and an invalid belief leads to adaptive behavior. In this way the fallacious belief is adaptive thinking in Gigerenzer's (2000) terms.

The hot hand is by no means the only phenomenon in which adaptive behavior may flow from a fallacious belief. Shelly Taylor and her colleague's work on the benefits of false optimism (e.g., Taylor & Brown, 1988, 1994) seems to be another example. The benefits of false optimism similarly illustrate the difference between analyzing a phenomenon in terms of the rationality of the belief versus the adaptiveness of the behavior. Anderson (1990) and Gigerenzer (2000) provide other examples.

There is no doubt that the beliefs people hold play an important role in their decision making and thus identifying those beliefs is useful. However, unless one thinks it is better to score less in basketball than would be possible if a simple cue was given some weight, then it appears that a research focus on belief without regard to behavior has lead to the mis-analysis of an important decision making phenomenon. In doing so this approach has obscured interesting aspects of a ubiquitous phenomenon, possibly discouraged research into it, and potentially has led to the mis-analysis of other phenomenon.

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Footnotes

[1]I will not be taking into account that often players will be passed to without the passer having the intention that they take the shot, or that once a player is passed to they might decide not to shoot. I am simply assuming that at some point a player with the ball makes a decision to take the shot or to pass it someone in a better position. In that sense they are allocating a shot, but there may be multiple such "allocations" made before a shot is actually taken. My analysis will only be in terms of shots taken.

[2]My analysis does not take into account higher-order choices such as deciding to continue or cease making choices. For example, following streaks could be harmful for gamblers not because of the options they bet on, but because it encourages them to keep playing the game. Deciding how much to allocate in total across options raises different issues to deciding how to allocate a fixed amount between options.

Table 1

Proportions of players' total shots that were parts of runs of 1, 2, or 3 hits, or 1, 2, or 3 misses. Correlations are between these proportions and the players' season long shooting percentage (all significant at $p < .01$*)*

Player	Shooting percentage	Total shots	3 misses	2 misses	1 miss	1 hit	2 hits	3 hits
Lionel Hollins	.46	371	.11	.25	.54	.46	.18	.07
Andrew Toney	.46	406	.08	.22	.53	.47	.19	.07
Caldwell Jones	.47	225	.09	.21	.52	.48	.16	.05
Clint Richardson	.50	206	.06	.16	.49	.51	.22	.10
Julius Erving	.52	836	.11	.23	.49	.51	.25	.12
Bobby Jones	.52	310	.06	.17	.47	.53	.25	.11
Steve Mix	.54	386	.06	.17	.46	.54	.25	.09
Maurice Cheeks	.56	292	.04	.13	.43	.57	.26	.11
Daryl Dawkins	.62	358	.02	.09	.38	.62	.31	.15
Correlations with shooting percentage:			-.80	-.87	-.99	.99	.95	.90

Table 2

Participants' answers to the hot hand belief and hand hot behavior questions.

		Hot hand belief	
		yes	no
Hot hand behavior	yes	659	124
	no	189	390

Table 3

Participants' answers to the hot hand belief and hand hot behavior questions as a function of their skill level.

		Skill level		
		low	medium	high
Hot hand belief	yes	326	323	199
	no	254	187	73
Hot hand behavior	yes	277	309	197
	no	303	201	75

Table 4

What participants who endorsed the hot hand belief thought was the implication of this belief as a function of their skill level.

		Skill level		
		low	medium	high
Implication	pass	199	226	163
	none	115	89	35
	don't pass	12	1	1

Table 5

What participants who endorsed the hot hand behavior thought was the explanation for this as a function of their skill level.

		Skill level		
		low	medium	high
Explanation	Hot hand	121	176	141
	Better player	130	114	39
	Both/other	26	18	17

Appendix

The expected number of hits after two shots can be calculated by adding up the expected outcomes of the 16 states in Figure 2. Parameter *b* represents the bias to give the ball to Player X, *h* represents belief in the hot hand, *x* stands for the shooting ability of Player X, and *y* stands for the shooting ability of Player Y.

$$
\begin{aligned}
E(hits) = \; & 2bx(b + h(1 - b))x + bx(b + h(1 - b))(1 - x) \\
& + 2bx(1 - b)(1 - h)y + bx(1 - b)(1 - h)(1 - y) \\
& + b^2(1-x)x + b(1-x)(1-b)y \\
& + 2(1 - b)yb(1 - h)x + (1 - b)yb(1 - h)(1 - x) \\
& + 2(1 - b)y(1 - b(1 - h))y + (1 - b)y(1 - b(1 - h))(1 - y) \\
& + (1 - b)(1 - y)bx + (1 - b)^2(1 - y)y
\end{aligned}
$$

$$
\begin{aligned}
E(hits) = \; & bx(b + h(1 - b))x + bx(b + h(1 - b)) \\
& + bx(1 - b)(1 - h)y + bx(1 - b)(1 - h) \\
& + xb^2 - x^2b^2 + by - bxy - b^2y + b^2xy \\
& + (1 - b)yb(1 - h)x + (1 - b)yb(1 - h) \\
& + (1-b)y(1-b(1-h))y + (1-b)y(1-b(1-h)) \\
& + bx - b^2x - bxy + b^2xy + (1-2b + b^2)(y - y^2)
\end{aligned}
$$

$$
\begin{aligned}
E(hits) = \; & (x + 1)bx(b + h(1 - b)) \\
& + (y+1)bx(1 - b)(1 - h) \\
& + xb^2 - x^2b^2 + by - bxy - b^2y + b^2xy \\
& + (x + 1)(1 - b)yb(1 - h) \\
& + (y + 1)(1-b)y(1-b(1-h)) \\
& + bx - b^2x - bxy + b^2xy + y - 2by + b^2y - y^2 + 2by^2 - b^2y^2
\end{aligned}
$$

$$E(hits) = (x + 1)((b^2x + bxh - b^2xh) + (yb - ybh - yb^2 + yb^2h))$$
$$+ (y + 1)((bx - b^2x - bxh + b^2xh) + (y - yb + ybh - yb + yb^2 - yb^2h))$$
$$+ xb^2 - x^2b^2 + by - bxy - b^2y + b^2xy + bx - b^2x - bxy + b^2xy + y - 2by + b^2y - y^2 + 2by^2 - b^2y^2$$

$$E(hits) = b^2x^2 + bx^2h - b^2x^2h + ybx - ybhx - yb^2x + yb^2hx$$
$$+ bxy - b^2xy - bxhy + b^2xhy + y^2 - y^2b + y^2bh - y^2b + y^2b^2 - y^2b^2h$$
$$+ b^2x + bxh - b^2xh + yb - ybh - yb^2 + yb^2h + bx - b^2x - bxh + b^2xh + y - yb + ybh - yb + yb^2 - yb^2h$$
$$+ xb^2 - x^2b^2 + by - bxy - b^2y + b^2xy + bx - b^2x - bxy + b^2xy + y - 2by + b^2y - y^2 + 2by^2 - b^2y^2$$

$$E(hits) = h(b^2(- x^2 + yx + xy + y - y^2 - x + x - y) + b(x^2 - yx - xy + x - x + y - y + y^2)$$
$$+ (b^2x^2 - x^2b^2) + (y^2b^2 - b^2y^2) + (b^2xy - yb^2x - b^2xy + b^2xy) + (b^2x - b^2x - b^2x + xb^2) + (yb^2 - yb^2 +$$
$$b^2y - b^2y) + (ybx + bxy - bxy - bxy) + (2by^2 - y^2b - y^2b) + (bx + bx) + (by + yb - yb - yb - 2by) +$$
$$(y + y) + (y^2 - y^2)$$

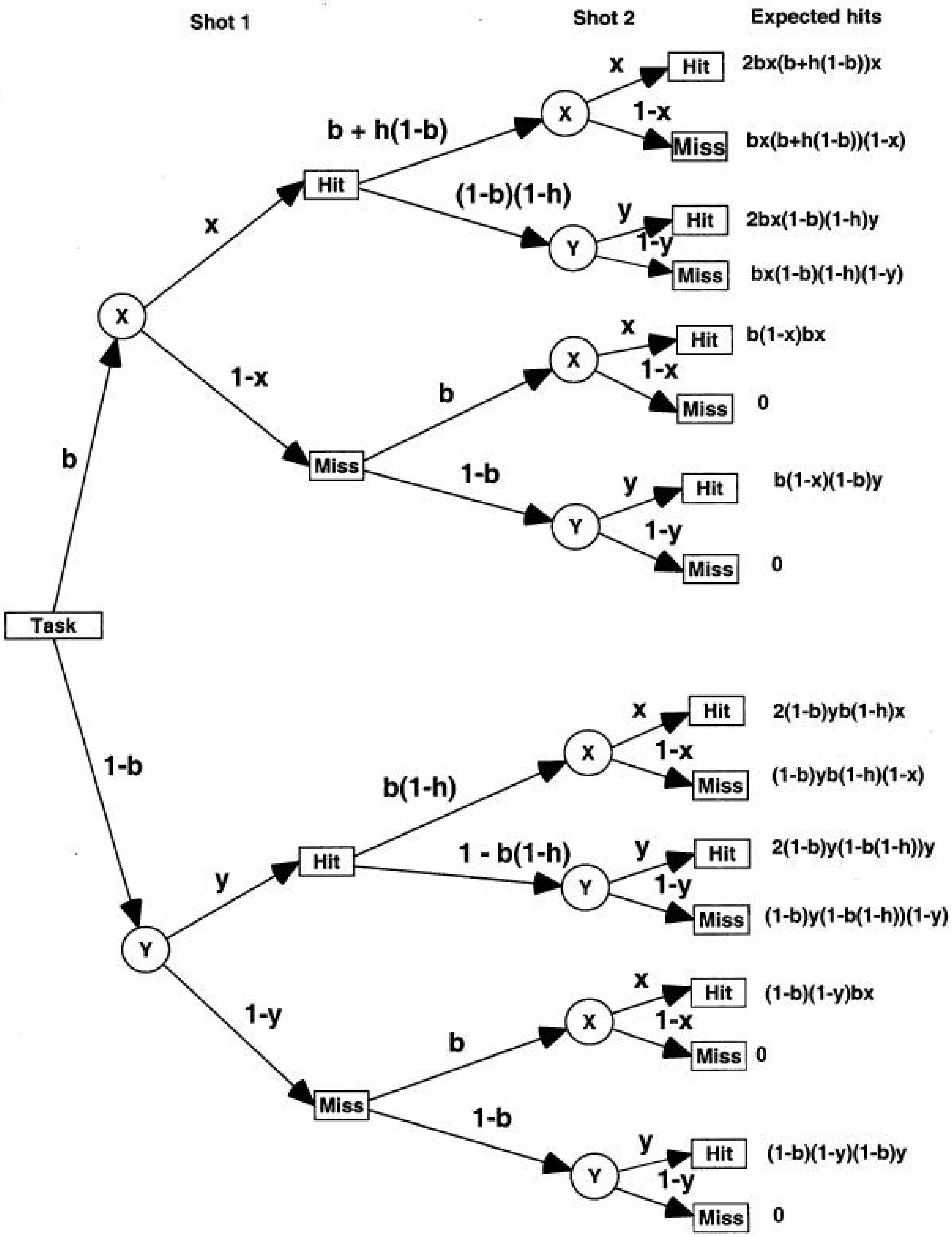
$$E(hits) = h(b^2(-x^2 + 2yx - y^2) + b(x^2 -2yx + y^2)) + 2bx - 2yb + 2y$$

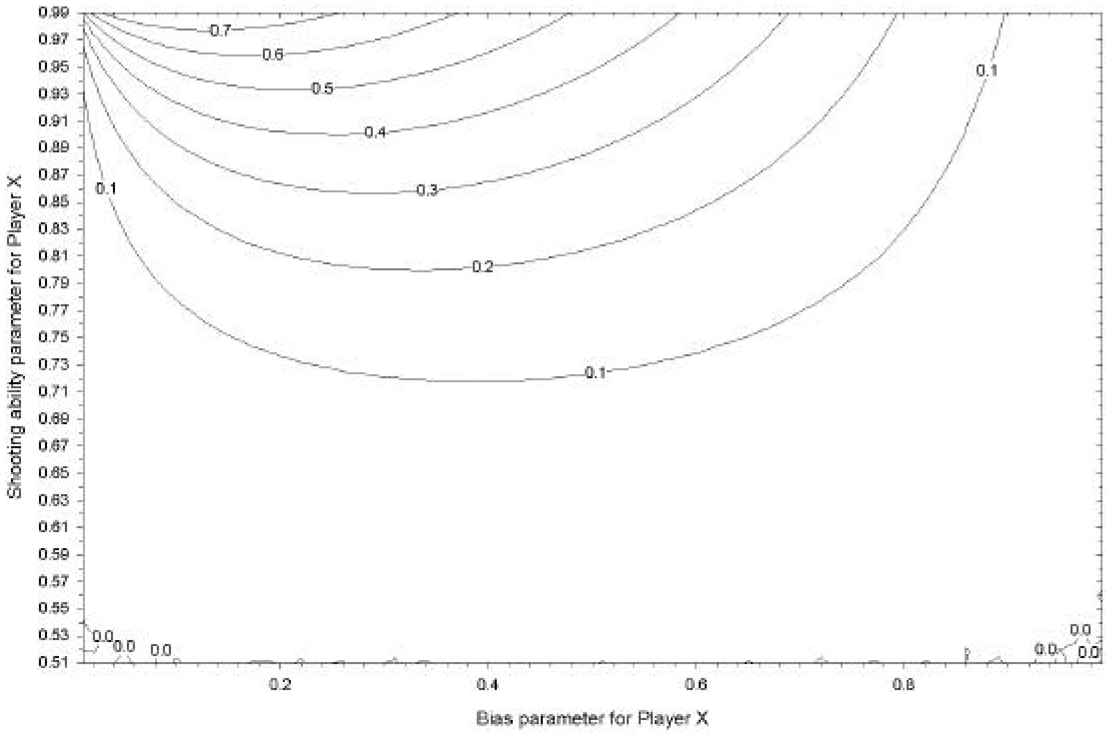
$$E(hit) = h(b - b^2)(x - y)^2 + 2(b(x - y) + y)$$

Figure Captions

Figure 1. A Markov model of the first two shots in a basketball game showing the expected number of hits for each of the 16 states. Circles represent who took the shot (Player X or Player Y) and rectangles represent to result of a shot (a hit or a miss). The four parameters of this model are *b* (bias to give the ball to Player X), *h* (belief in the hot hand), *x* (representing the probability of Player X hitting a shot), and *y* (representing the probability of Player Y hitting a shot).

Figure 2. Contour graph plot of the advantage scores in favor of the hot hand simulation for each combination of shooting ability and bias parameters. Player X is assumed to have the higher shooting ability.


Shot 1
Shot 2
Expected hits
Task
b
1-b
X
Y
x
1-x
y
1-y
Hit
Miss
b + h(1-b)
(1-b)(1-h)
b
1-b
b(1-h)
1 - b(1-h)
2bx(b+h(1-b))x
bx(b+h(1-b))(1-x)
2bx(1-b)(1-h)y
bx(1-b)(1-h)(1-y)
b(1-x)bx
0
b(1-x)(1-b)y
0
2(1-b)yb(1-h)x
(1-b)yb(1-h)(1-x)
2(1-b)y(1-b(1-h))y
(1-b)y(1-b(1-h))(1-y)
(1-b)(1-y)bx
0
(1-b)(1-y)(1-b)y
0


Shooting ability parameter for Player X
Bias parameter for Player X
0.99
0.97
0.95
0.93
0.91
0.89
0.87
0.85
0.83
0.81
0.79
0.77
0.75
0.73
0.71
0.69
0.67
0.65
0.63
0.61
0.59
0.57
0.55
0.53
0.51
0.2
0.4
0.6
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0