Does the law of small numbers explain the gambler's fallacy?

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Summary

- Law of small numbers has been used to explain *Gambler's fallacy*, but under what conditions will it predict people to go against streaks?
- Experiment described sequences of coin-flips, used unequal base-rate, and varied streaks and generating mechanism (random vs nonrandom)
- No evidence of gambler's fallacy for random mechanism, and nonrandom mechanism seemed to create induction task.

Theoretical background

Law of small numbers

- Belief in the law of small numbers (LSN) leads people to expect every segment of a random sequence to reflect the true proportions (Tversky & Kahneman, 1971).
- Used to explain *Gambler's fallacy*: a tendency for people to expect a streak of one event to be followed by a different event.

Problems with this Law

- What is a segment?
  - •Most current streak of identical events?
  - •Frequency of events over some arbitrarily small set of events?
- Can the size of the segment change for the same task?
- Without some answers to these questions, it is difficult to use the law of small numbers to make predictions.

Randomness

- Burns & Corpus (in press) showed that participants' interpretation of streaks varied with the *generating process*.
  - •If process was random (e.g., coin flip), then participants more likely expect streak to *stop*.
  - •If process was nonrandom (e.g., competition), then participants more likely to expect streak to *continue*.

A study

Aims

Contrast streak length and frequency within a segment of events.

- Present participants with reported sequences of *eight* coin flips, but *five* had been HEADS.
- Sequence always ended with TAIL, but other two tails appeared in all possible positions (21 combinations) so that some sequences contain streaks.
- Also varied generating mechanism: random vs. nonrandom

Predictions

•If LSN applies to whole sequence, then participants should favor TAILS consistently.

•If LSN applies to streaks, then frequency of TAILS should be product of specific sequence.

•If mechanism nonrandom, then should favor HEADS (higher base-rate) overall, but favor TAILS when streak present (i.e., pairs of TAILS).

Materials

Random (for specific sequence **TT**HHHH**T**):

Imagine that you observed a sequence of eight coin flips. The sequence was as follows:

1. TAIL
2. TAIL
3. HEAD
4. HEAD
5. HEAD
6. HEAD
7. HEAD
8. TAIL

If you had to bet on the result of the next coin flip, what would you choose?

HEAD TAIL

Nonrandom - Replace first sentence with:

Imagine that you observed a sequence of eight "coin flips" generated by a computer program. However, you know that the program contains an error and may not be generating truly random "flips."

Participants: 648 members of the MSU subject pool. Each received only one sequence, either with the random or the nonrandom mechanism.

Results

Results for each sequence

	Random		Nonrandom	
	HEAD	TAIL	HEAD	TAIL
HHHHH**TTT**	8	8	5	9
HHHH**T**H**TT**	6	10	7	9
HHHH**TT**H**T**	7	7	7	7
HHH**T**HH**TT**	7	9	10	5
HHH**T**HTH**T**	9	7	12	4
HHH**TT**HH**T**	9	7	11	5
HH**T**HHH**TT**	8	6	12	4
HH**T**HH**T**H**T**	9	6	12	4
HH**T**HTHH**T**	7	9	14	2
HH**TT**HHH**T**	3	11	5	11
H**T**HHHH**TT**	8	8	10	6
H**T**HHH**T**H**T**	9	7	14	2
H**T**HH**T**HH**T**	6	9	13	2
H**T**H**T**HHH**T**	9	6	10	6
H**TT**HHHH**T**	7	8	8	8
THHHHH**TT**	4	11	11	5
THHHH**T**H**T**	8	7	13	3
THHH**T**HH**T**	9	6	13	3
THH**T**HHH**T**	6	7	15	1
THT**T**HHHH**T**	8	7	13	3
TTHHHHH**T**	6	9	5	10
Total	153	165	220	109

For Random mechanism

•No difference in selecting HEAD or TAIL, $z = 0.61$.

•No effect of sequence, $X^2(20) = 13.2$, $p = .87$.

For Nonrandom mechanism

•HEAD selected more than TAIL, $z = 6.06$.

•Effect of sequence, $X^2(20) = 50.8$, $p < .001$

Sequences with pairs of Tails:

	Random		Nonrandom	
	Heads	Tails	Heads	Tails
No pairs	113	115	179	59
Pairs	40	50	41	50

- For random mechanism, no overall effect of pairs of TAILS, $X^2(1) = 0.68$, $p = .41$.
- For nonrandom mechanism, relatively more likely to choose TAILS when there was a pair of TAILS, $X^2(1) = 27.0$, $p < .001$

Why no evidence here for Gambler's fallacy?

•Surprisingly, no evidence of Gambler's fallacy in any condition.

•Maybe base-rate favoring HEADS and fact that last item was TAIL cancelled each other out. But exactly?

•Perhaps Gambler's fallacy is fragile and will only apply in the absence of any other usable information.

•Perhaps fallacy only applies when sequence is experienced one item at a time.

•But many sequences are presented as history, and some report Gambler's fallacy behavior (e.g., lotteries).

Conclusions

•The mechanism by which a sequence is generated is critical to how it is interpreted.

•When mechanism is *nonrandom*, this seems to produce an induction task: base-rates and streaks are utilized.

•When mechanism is *random* no information has an impact: base-rates and streaks ignored.

•Law of small numbers had no predictive power in this experiment.

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References:

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Kahneman, D., & Tversky, A., (1971)Belief in the law of small numbers. *Psychological Bulletin, 2,* 105-110.